

# Underwater sound propagation within ocean general circulation

**ECCO Annual Meeting**

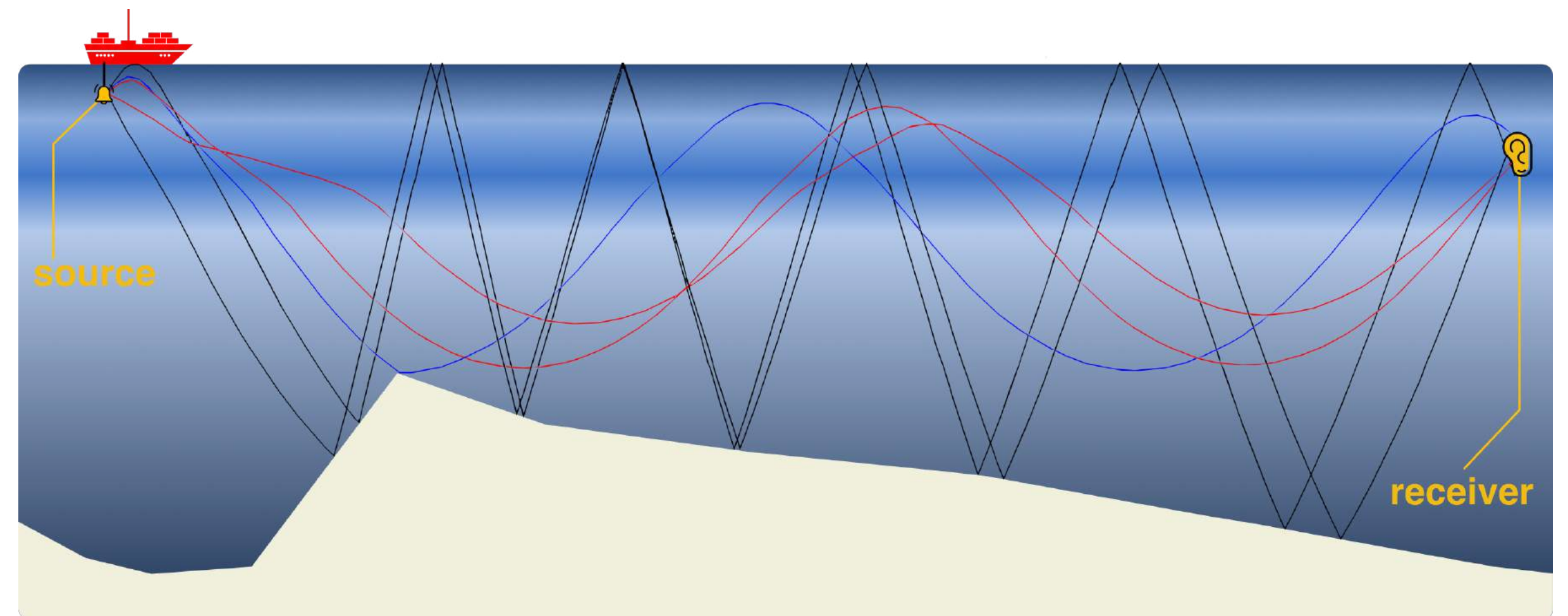
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**21 March 2024  
Austin, TX**

# Underwater acoustic observations

Tomography offers along path information of the ocean interior

- Active sonar pulses with known source and receiver positions can provide measurements untapped by ocean state estimates.
- Measured travel times are commonly compared with ray tracing modeled travel times.<sup>[ES09]</sup>
- Acoustic remote sensing can be added to state estimates as a new constraint in model-data misfit.
- Need a way to access acoustic diagnostics *within* a general circulation model.



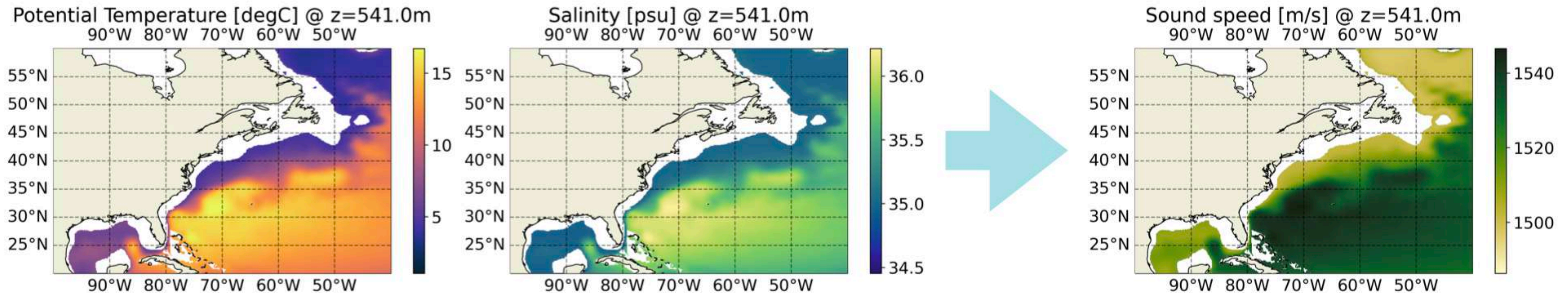


# Modeled sound speed

## Sample diagnostic results

- From modeled hydrography, sound speed is resolved as a diagnostic at each time step. The three-dimensional field is solved from the UNESCO empirical algorithm.<sup>[CC77]</sup>

$$c(S, T, p) = C_w(T, p) + A(T, p)S + B(T, p)S^{2/3} + D(T, p)S^2 .$$

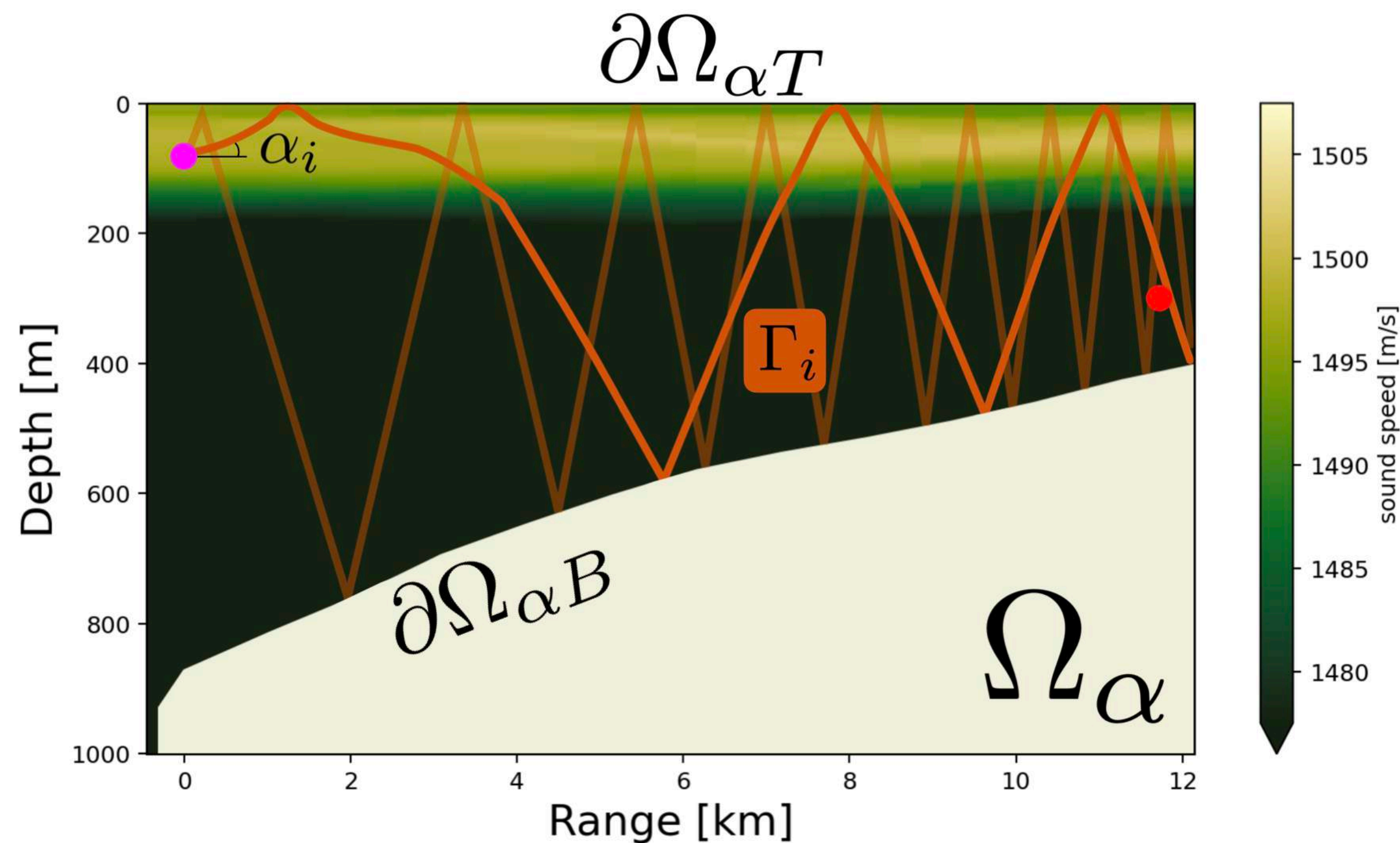




# Geometric ray trace model

## Two-dimensional range-dependent equations

Let  $\mathbf{r} = (r, z) \in \Omega_\alpha$ ,  $\gamma \in \Omega_\alpha$ , is an arc length along a ray,  $\Gamma(\gamma)$ . In ray-centered coordinates,  $\xi(\gamma)$  and  $\zeta(\gamma)$ , the tangent vector is  $\mathbf{t} = c[\xi, \zeta]$ .<sup>[FJ11]</sup>



$$\frac{dr}{d\gamma} = \mathbf{t} \quad \text{in } \Omega_\alpha,$$

$$\frac{dt}{d\gamma} = -\frac{1}{c} \nabla_r c \quad \text{in } \Omega_\alpha,$$

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$$\mathbf{t} = \mathbf{t}_T \quad \text{on } \partial\Omega_{\alpha T},$$

$$\mathbf{t} = \mathbf{t}_B \quad \text{on } \partial\Omega_{\alpha B},$$

$$\nabla \mathbf{r} \cdot \mathbf{n} = \mathbf{0} \quad \text{on } \partial\Omega_{\alpha R},$$

$$\mathbf{r} = \mathbf{r}_0 \quad \text{on } \partial\Omega_{\alpha S},$$

$$\mathbf{t} = \mathbf{t}_0 \quad \text{on } \partial\Omega_{\alpha S}.$$

# Dynamic acoustic modeling

## Determining eigenrays seen at a receiver location

- For each  $i = 1, \dots, N_\alpha$ , given a launch angle  $\alpha_i$ , we obtain a set of paths,  $\Gamma_i = \{\mathbf{r}(\gamma) \in \Omega_\alpha : i = 1, \dots, N_\alpha\}$ . Travel times,  $\tau_i$ , are found along  $\Gamma_i$ .

$$\begin{array}{llll}
 \frac{d\tau_i}{d\gamma} = \frac{1}{c} & \text{in } \Gamma_i, & A = |\mathcal{R}|A & \text{on } \partial\Omega_{\alpha B}, \\
 \frac{dq_i}{d\gamma} = cp_i & \text{in } \Gamma_i, & \tau_i = 0 & \text{on } \Gamma_i \cap \partial\Omega_{\alpha S}, \\
 \frac{dp_i}{d\gamma} = -\frac{\nabla(\nabla c \cdot \mathbf{n}) \cdot \mathbf{n}}{c^2} q_i & \text{in } \Gamma_i, & q_i = 0 & \text{on } \Gamma_i \cap \partial\Omega_{\alpha S}, \\
 & & p_i = \frac{1}{c_0} & \text{on } \Gamma_i \cap \partial\Omega_{\alpha S},
 \end{array}$$

where  $p$ ,  $q$ , and  $|\mathcal{R}| \in [0,1]$  describe dynamic ray-centered coordinate, slowness, and Rayleigh reflection coefficient.

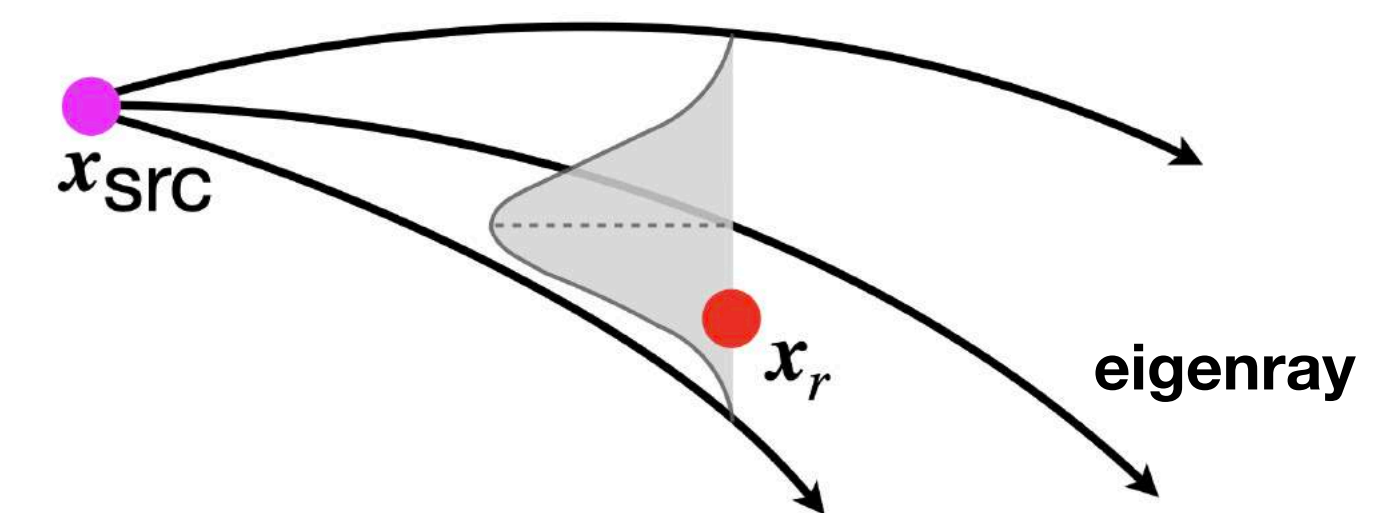
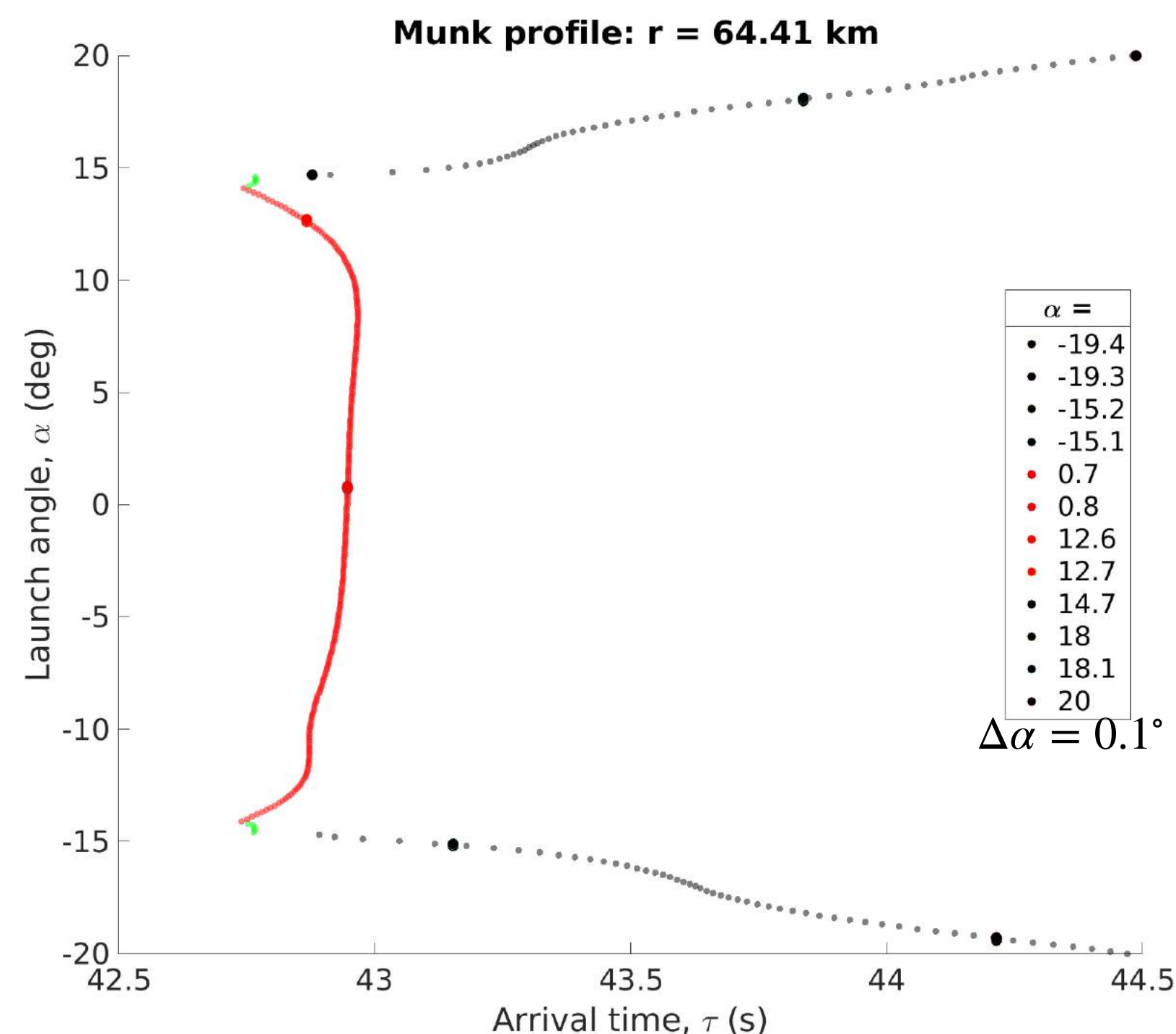
# Geometric ray trace model

## Determining eigenrays seen at a receiver location

- Geometric beams with Gaussian spreading projected onto the vertical direction from the ray path,  $\Gamma_i$ , as

$$W(n) = \exp\{-0.5(n(\gamma)/|q(\gamma)\delta\alpha|)^2\}$$

where  $n(\gamma)$  is the normal distance from receiver to the ray path and  $|q(\gamma)\delta\alpha|$  is a *beam halfwidth*.

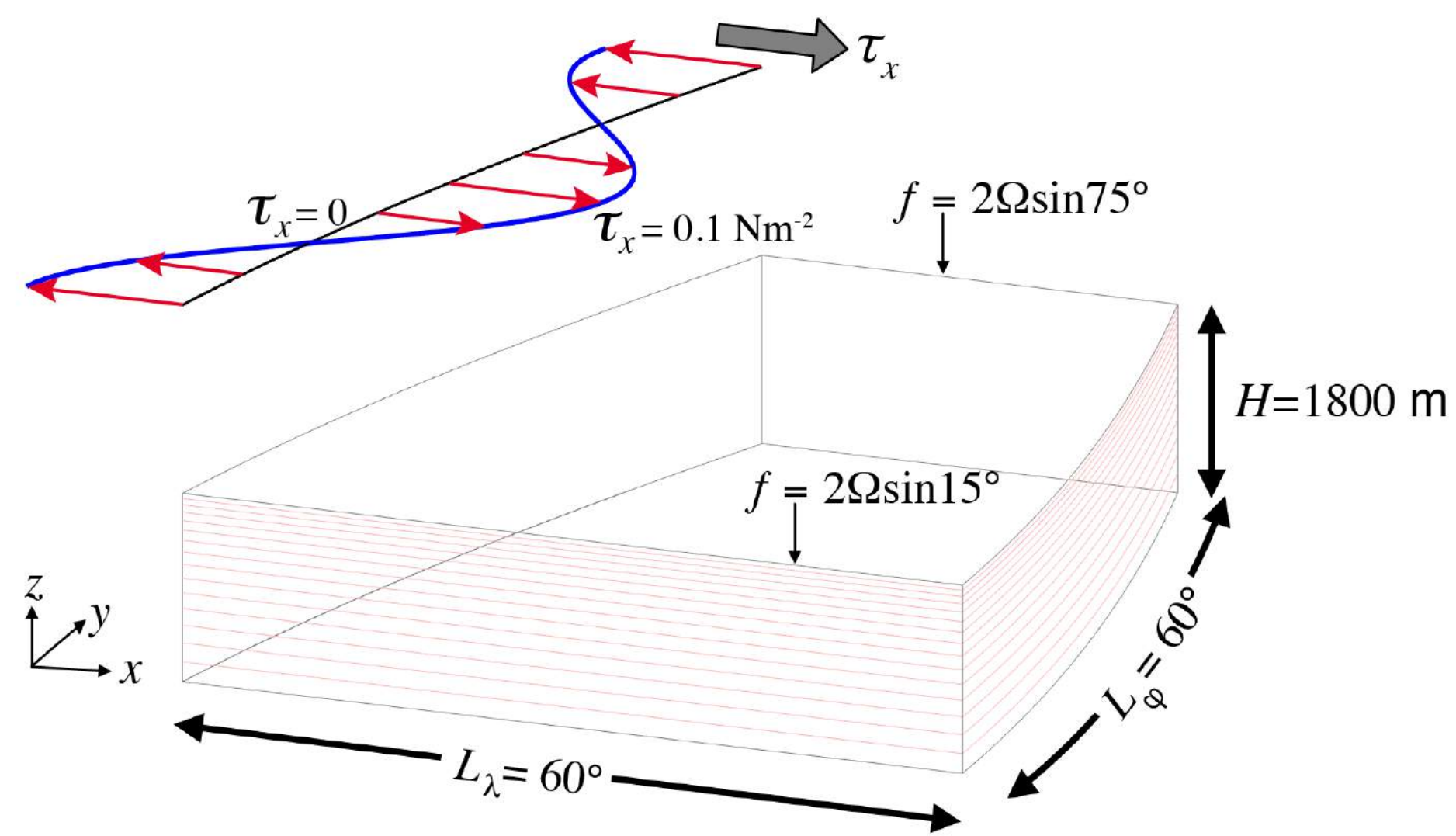


Adapted from Figure 3.9 in Jensen *et al.* (2011)

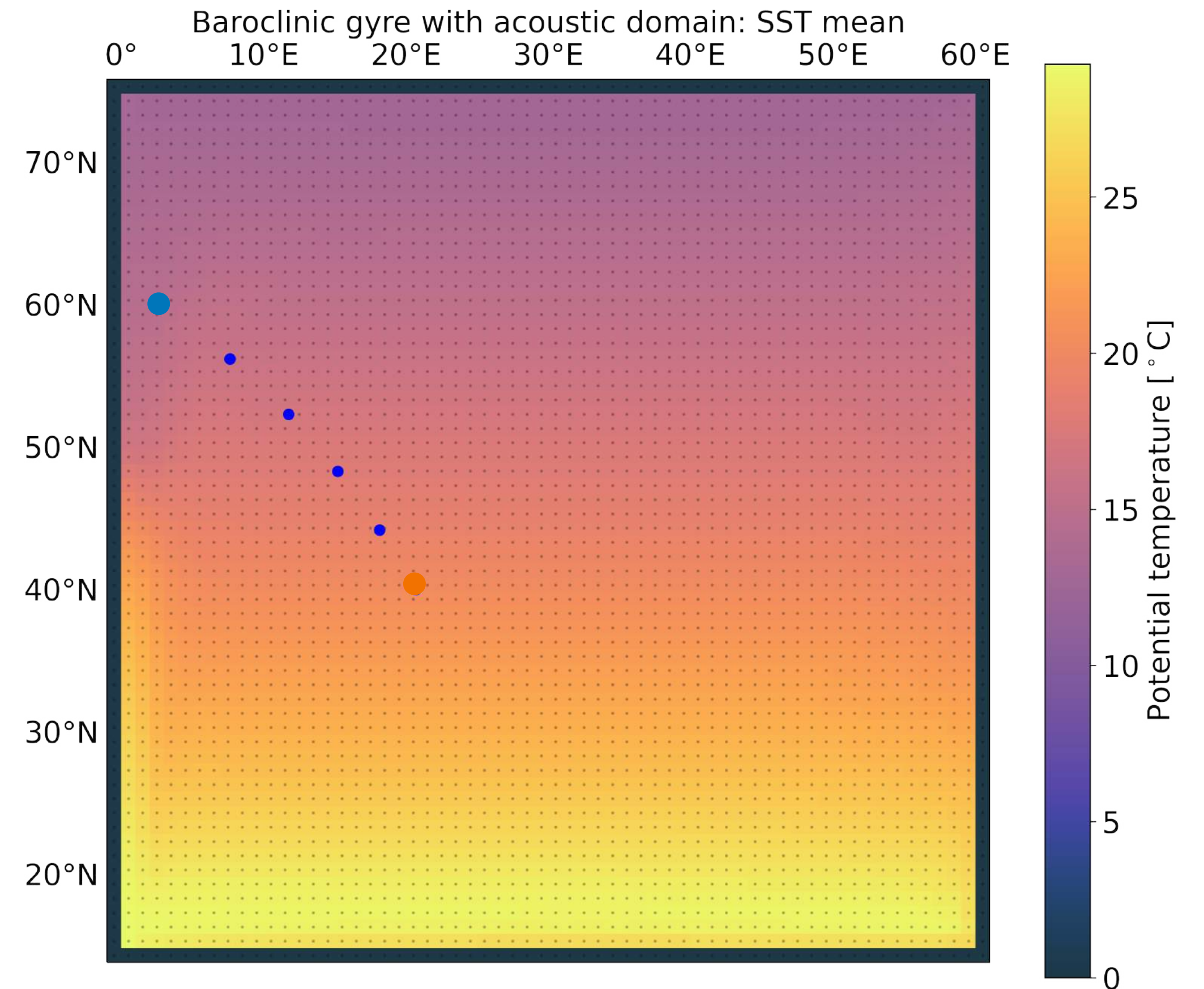


# Acoustic diagnostics from evolving ocean model

Results from a modeled baroclinic gyre<sup>[MC84]</sup>



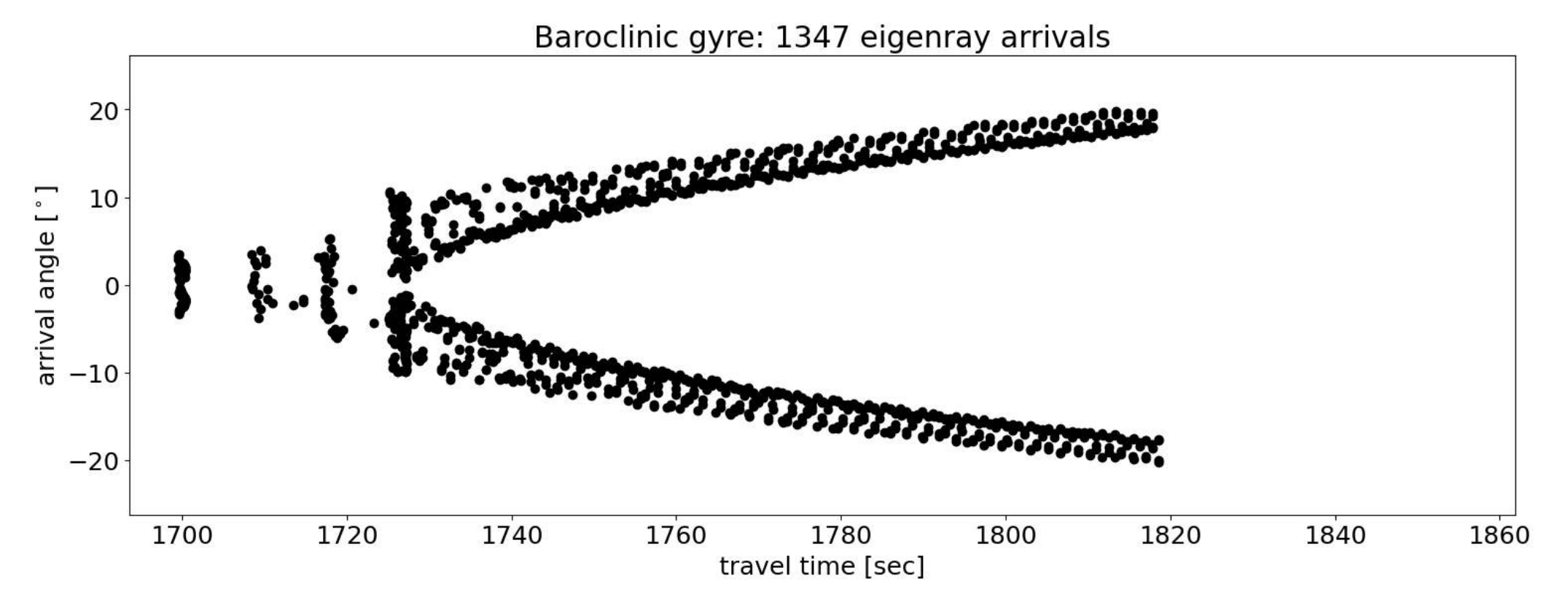
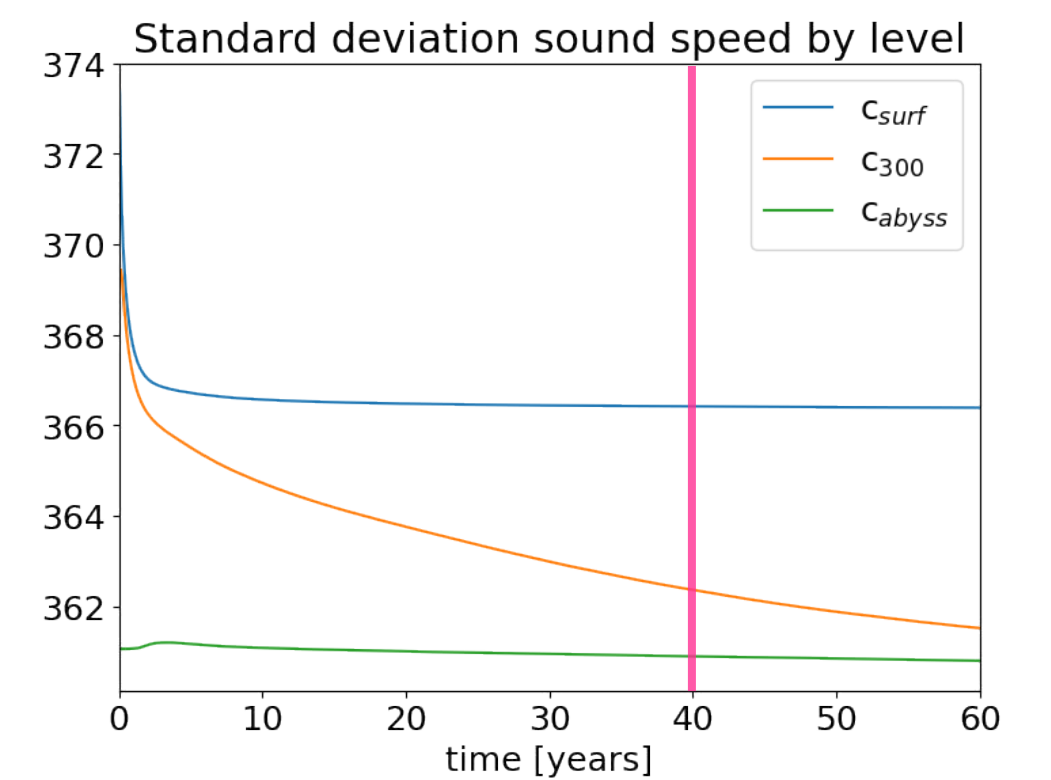
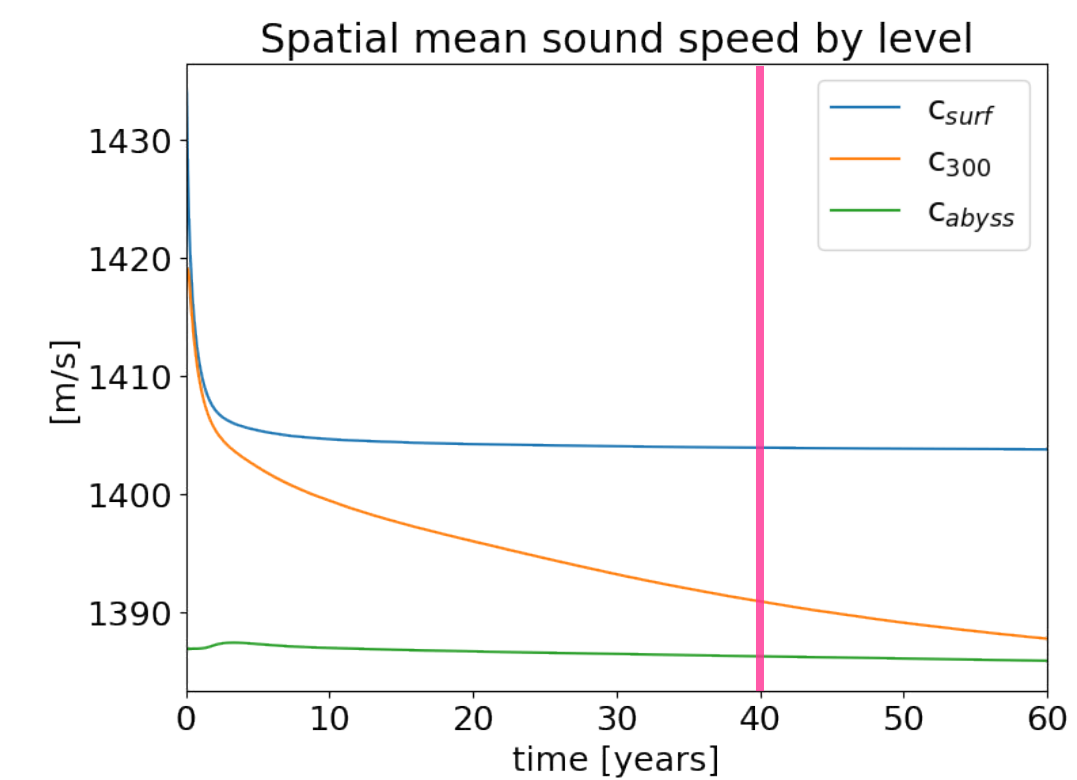
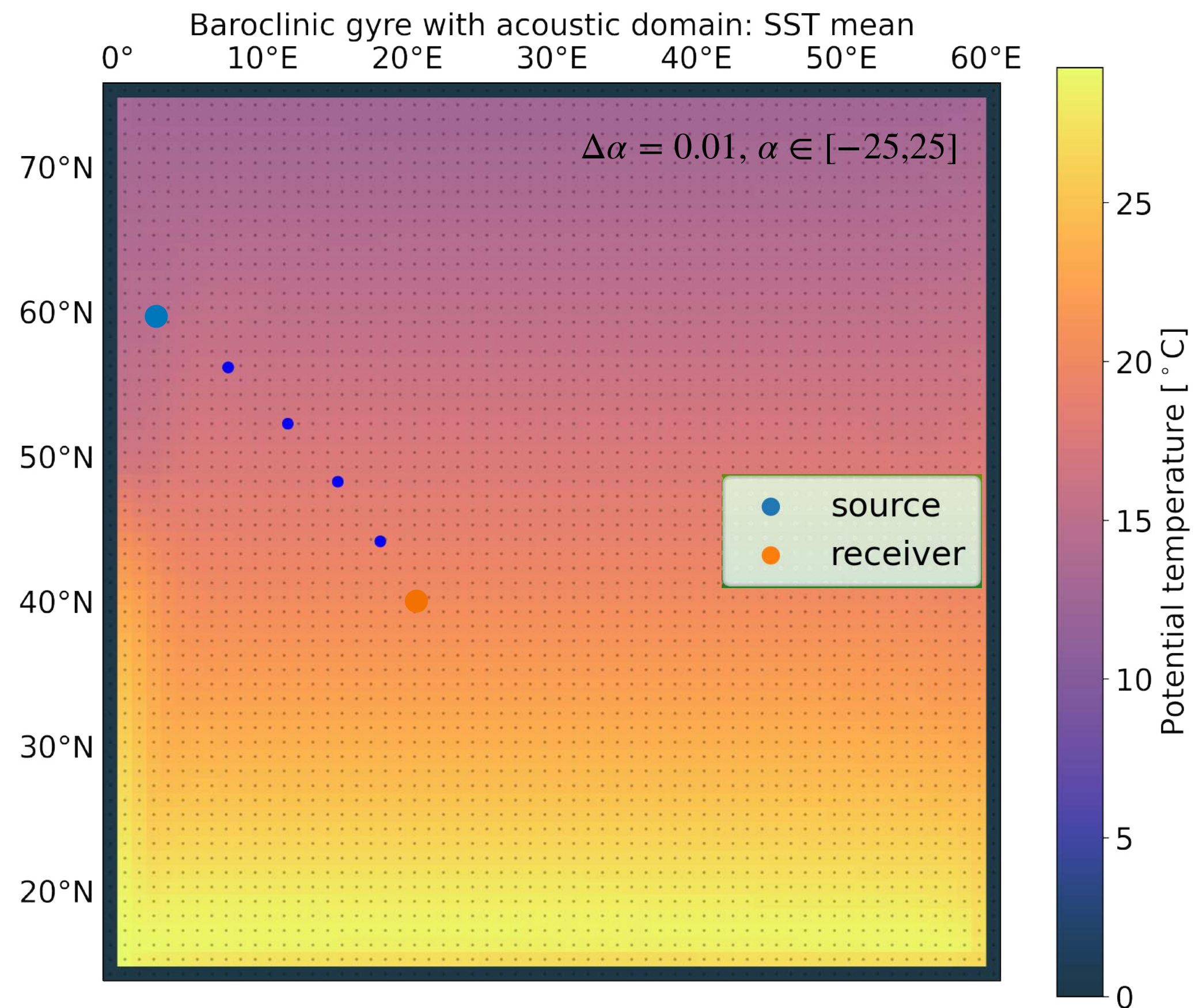
$$\tau_x(\varphi) = -\tau_0 \cos\left(2\pi \frac{\varphi - \varphi_0}{L_\varphi}\right)$$





# Acoustic diagnostics from evolving ocean model

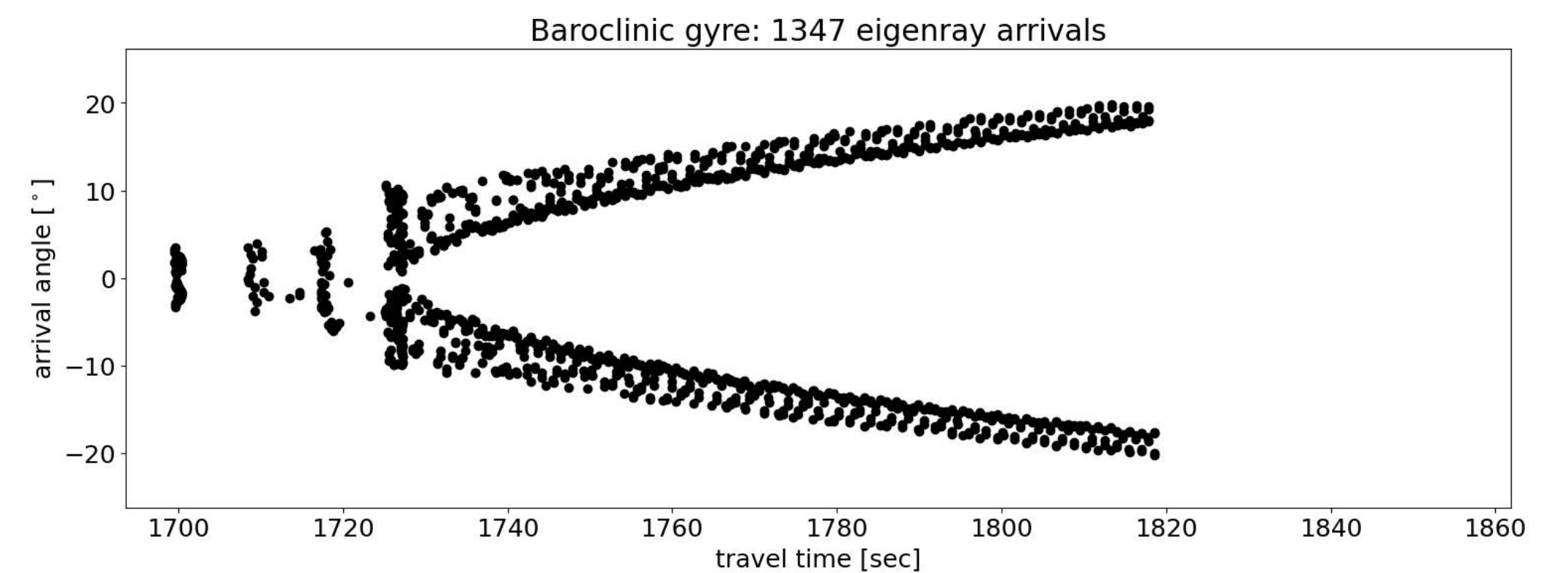
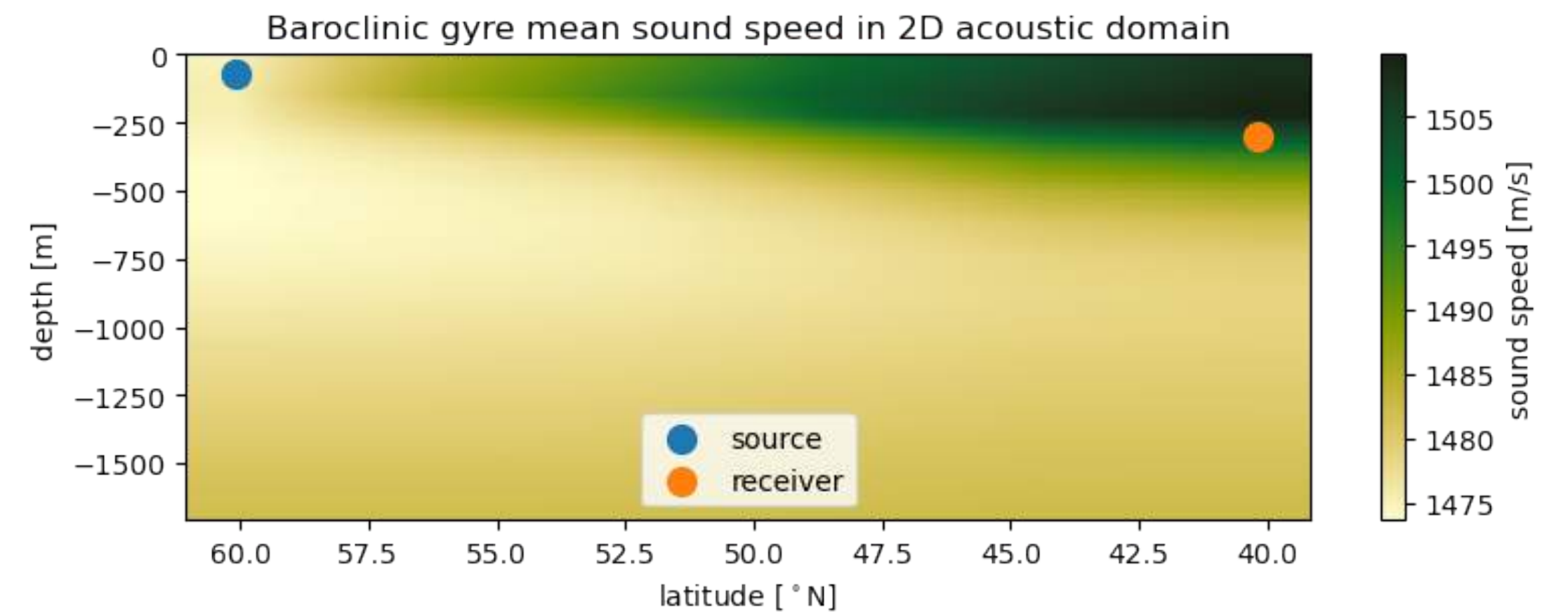
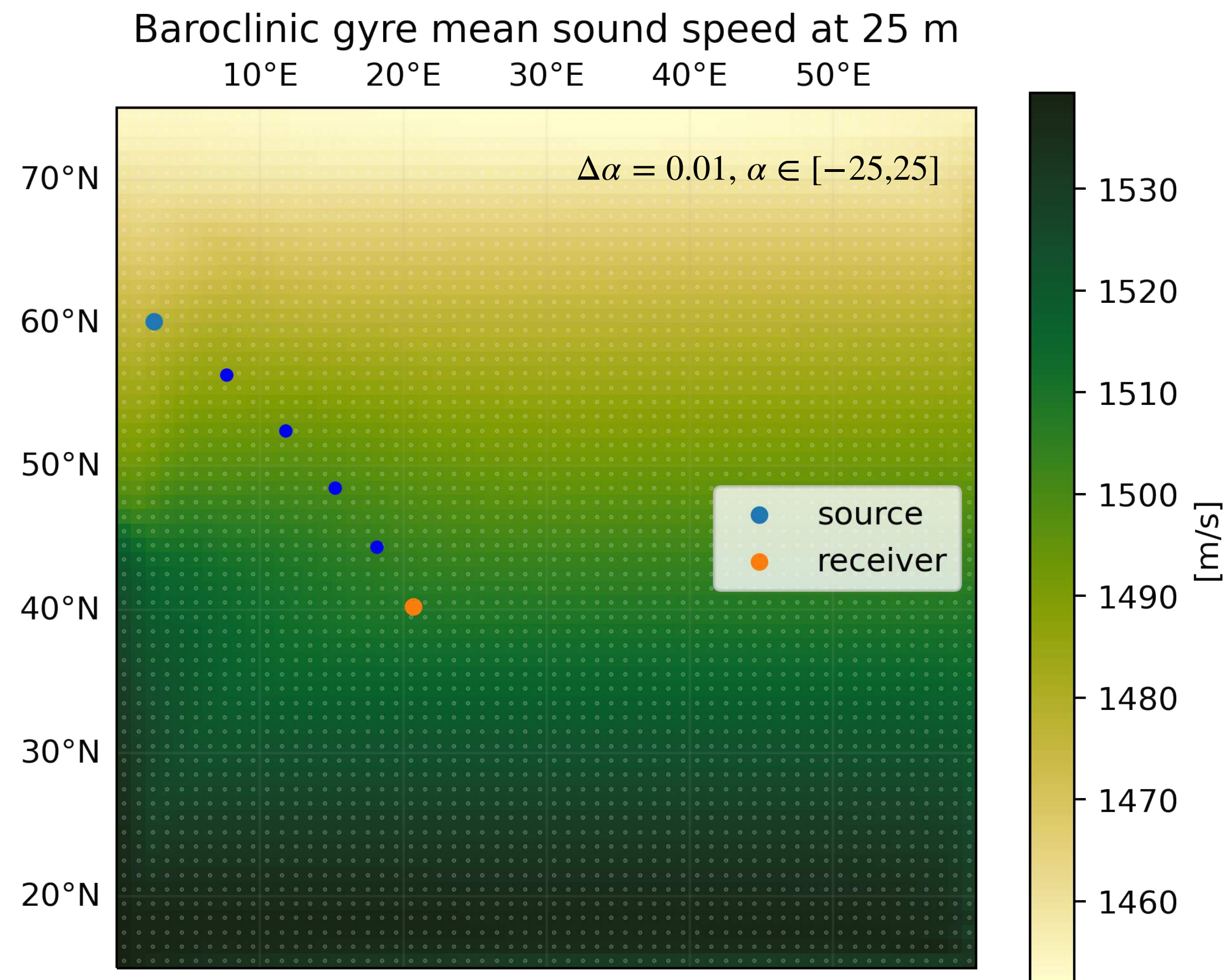
## Results from a modeled baroclinic gyre





# Acoustic diagnostics from evolving ocean model

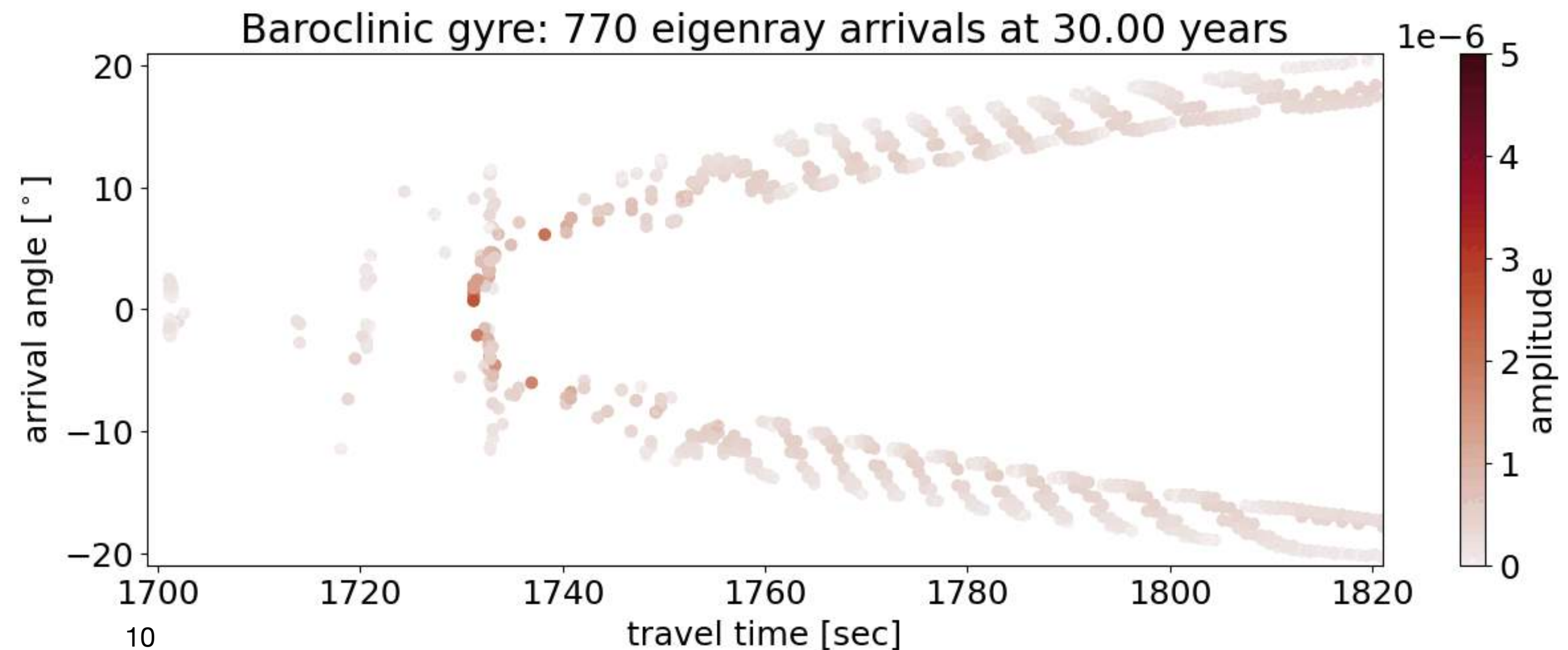
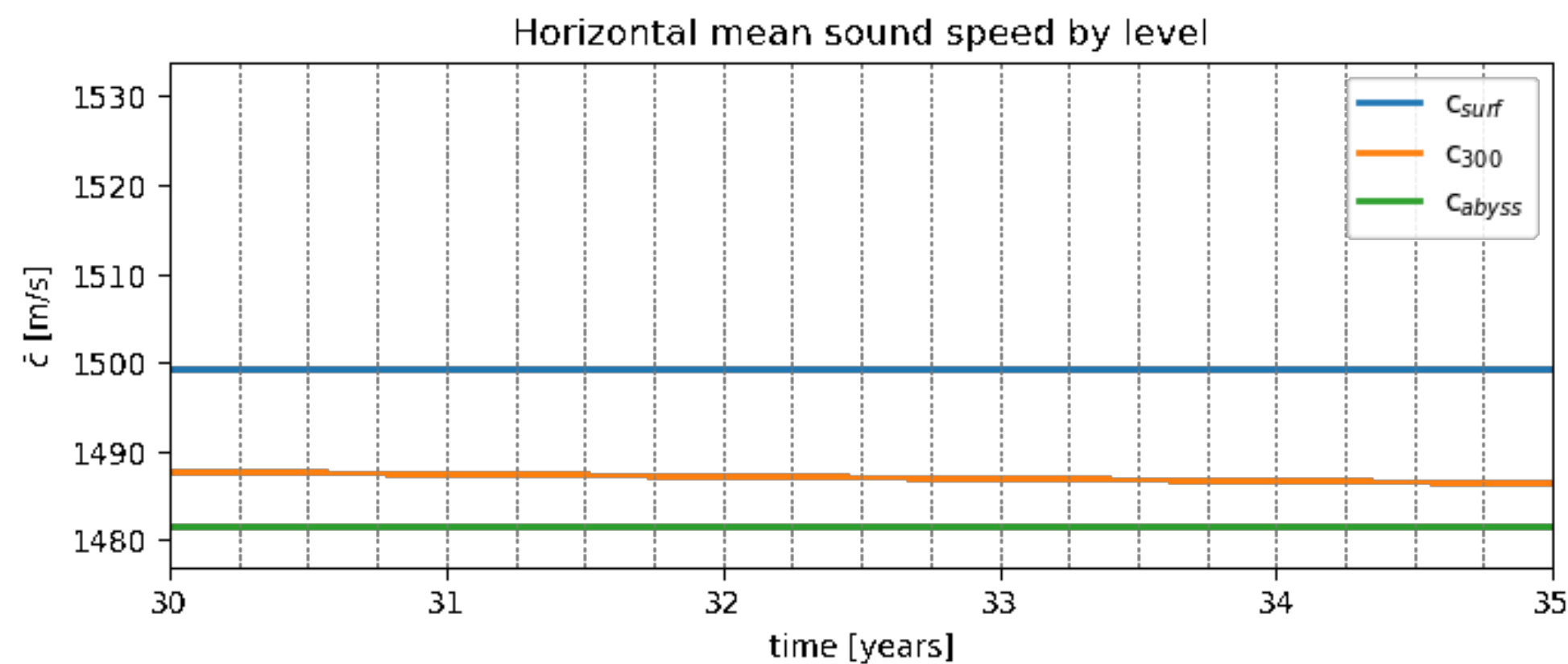
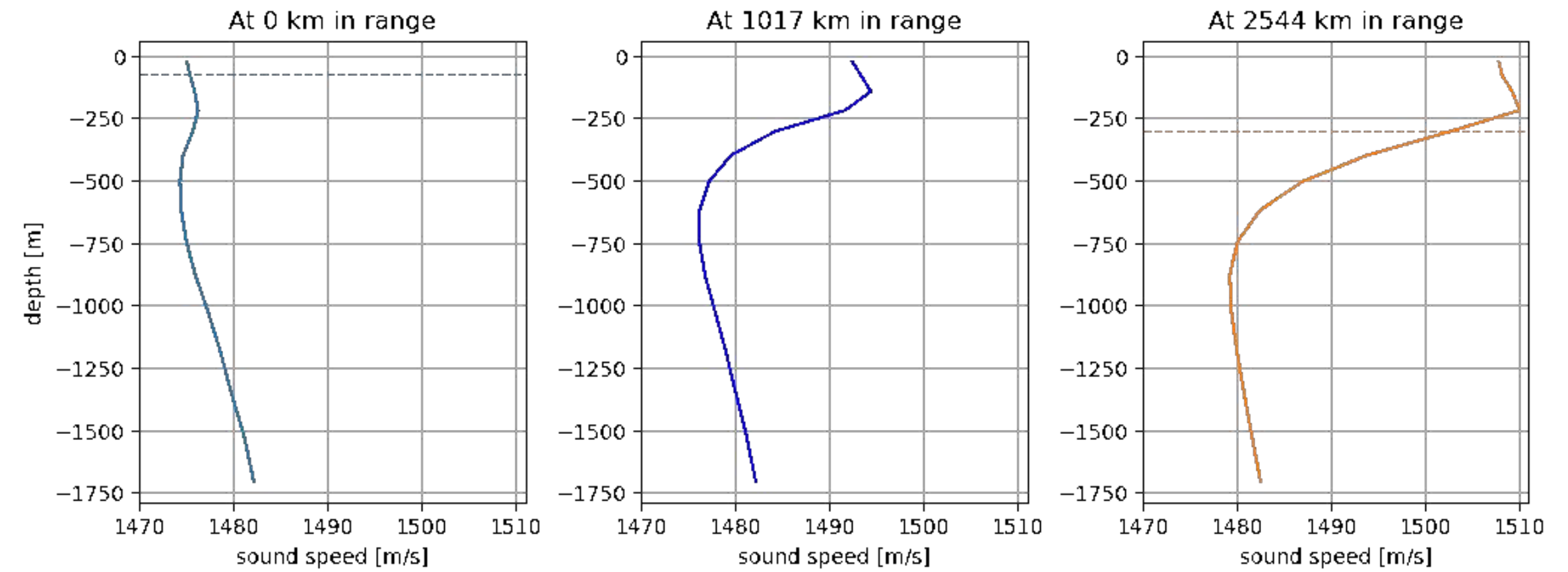
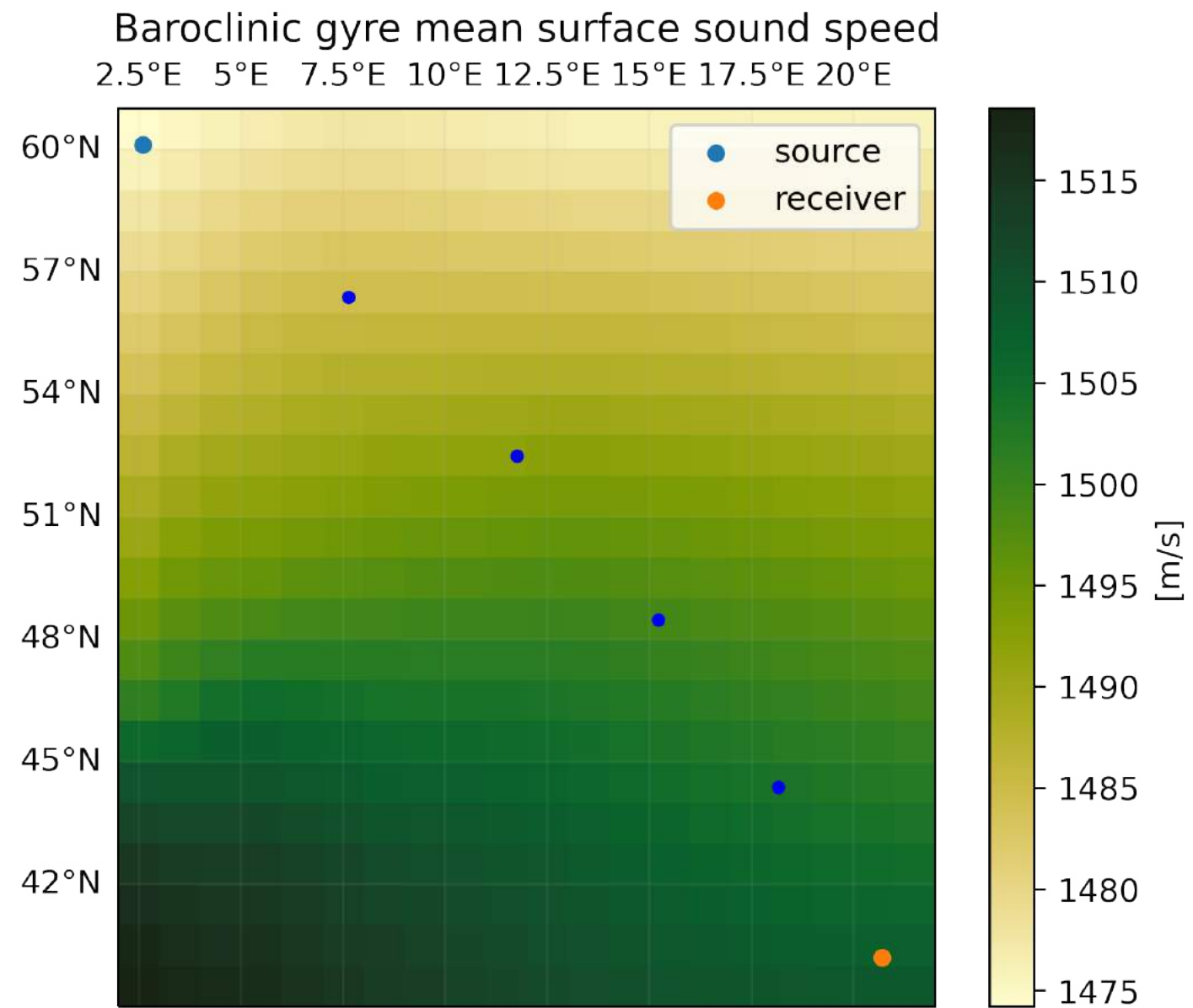
## Results from a modeled baroclinic gyre





# Acoustic diagnostics from evolving ocean model

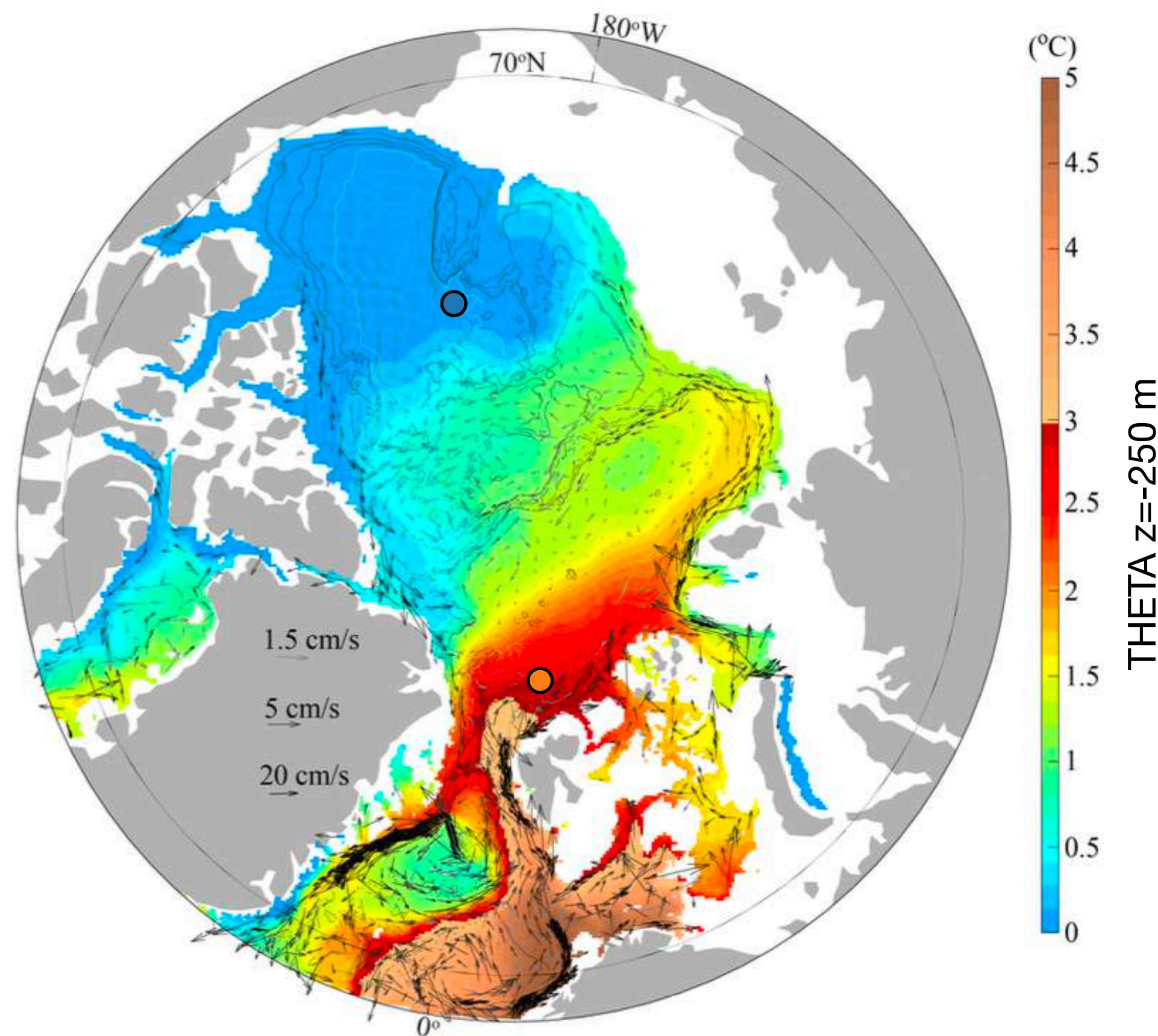
## Results from a modeled baroclinic gyre



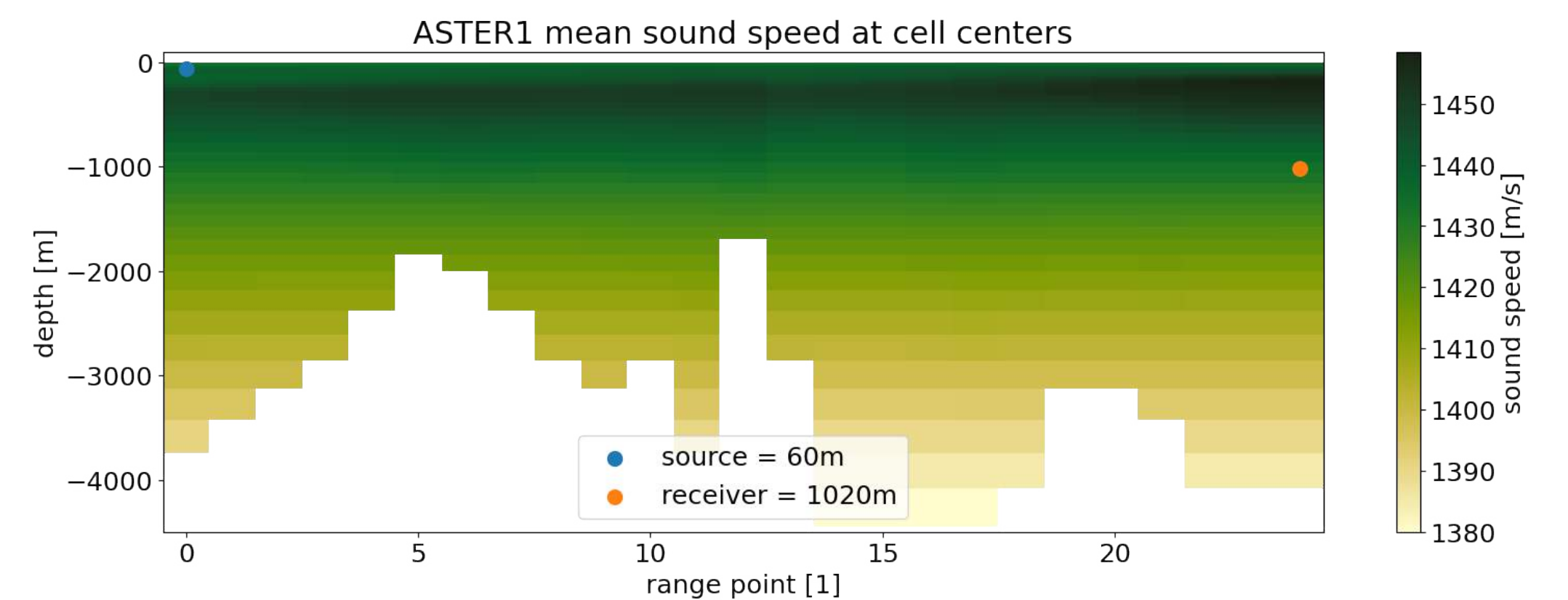
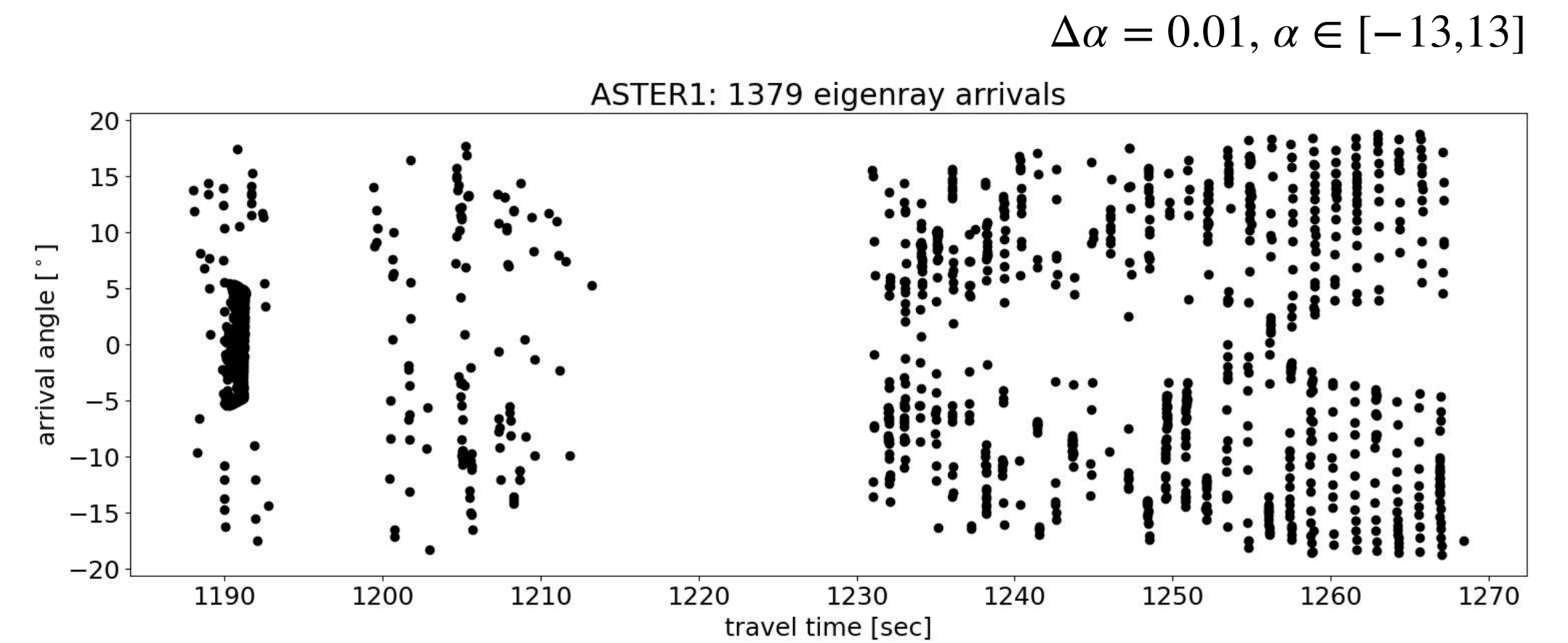


# Acoustic diagnostics from evolving ocean model

## Results on a *lat lon cap* ocean grid



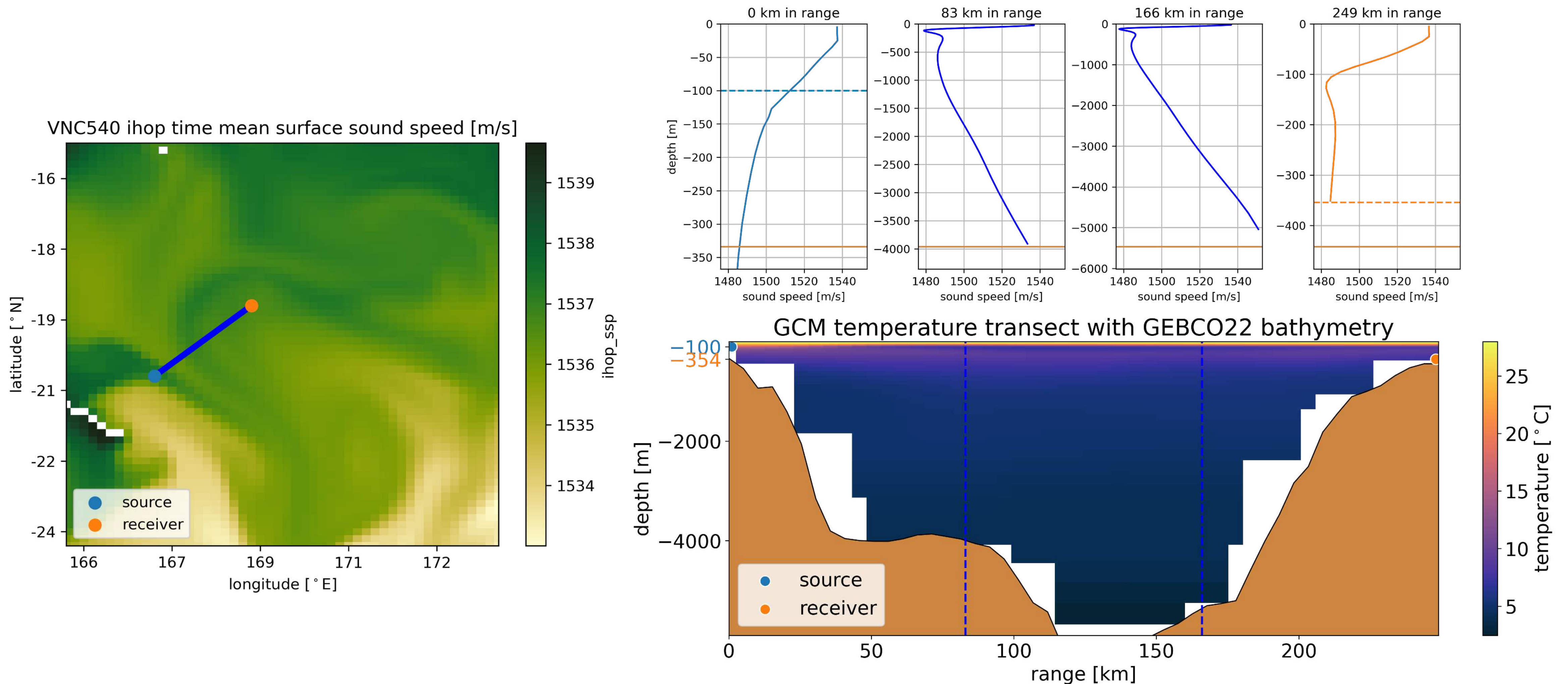
From Figure 2 in Nguyen *et al.* (2021)  
ASTER R1: 2002-2015 mean temperature





# Case study: Vanuatu/New Caledonia

Region developed by Matt Goldberg (tune in to upcoming talk)





# Case study: Vanuatu/New Caledonia

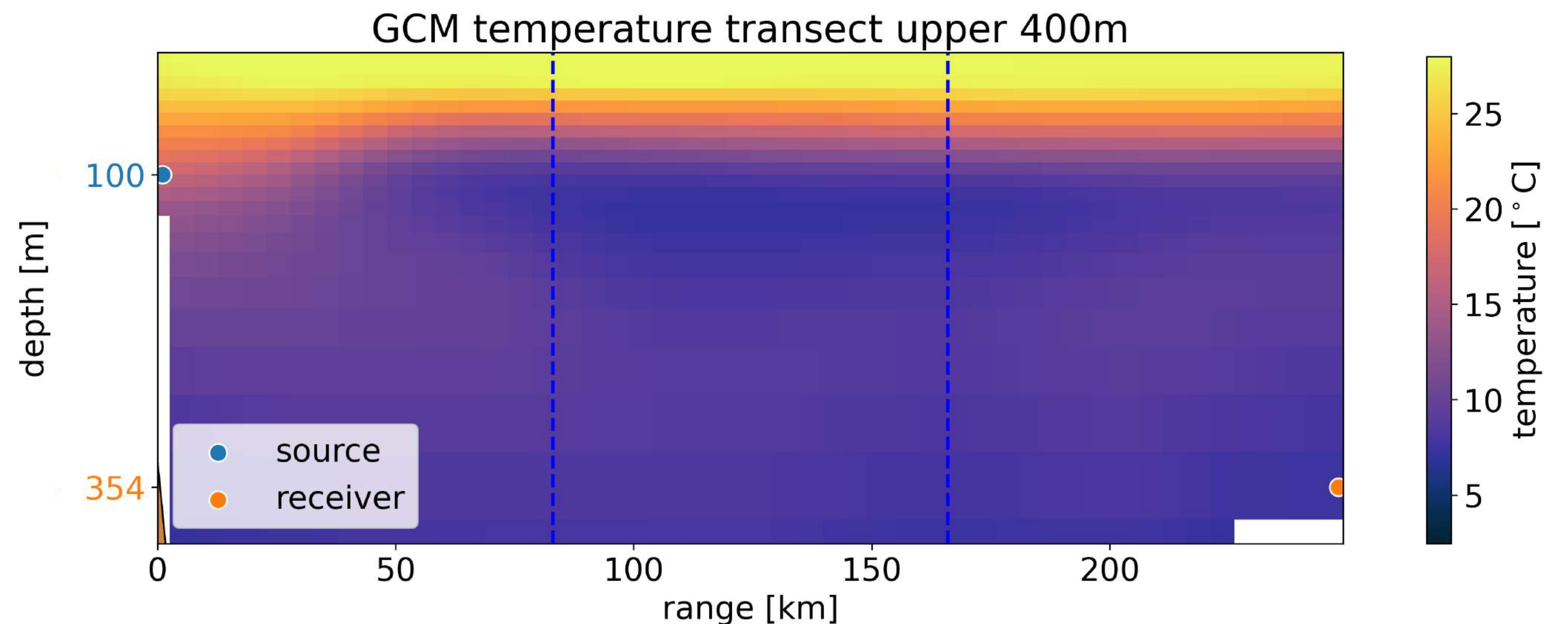
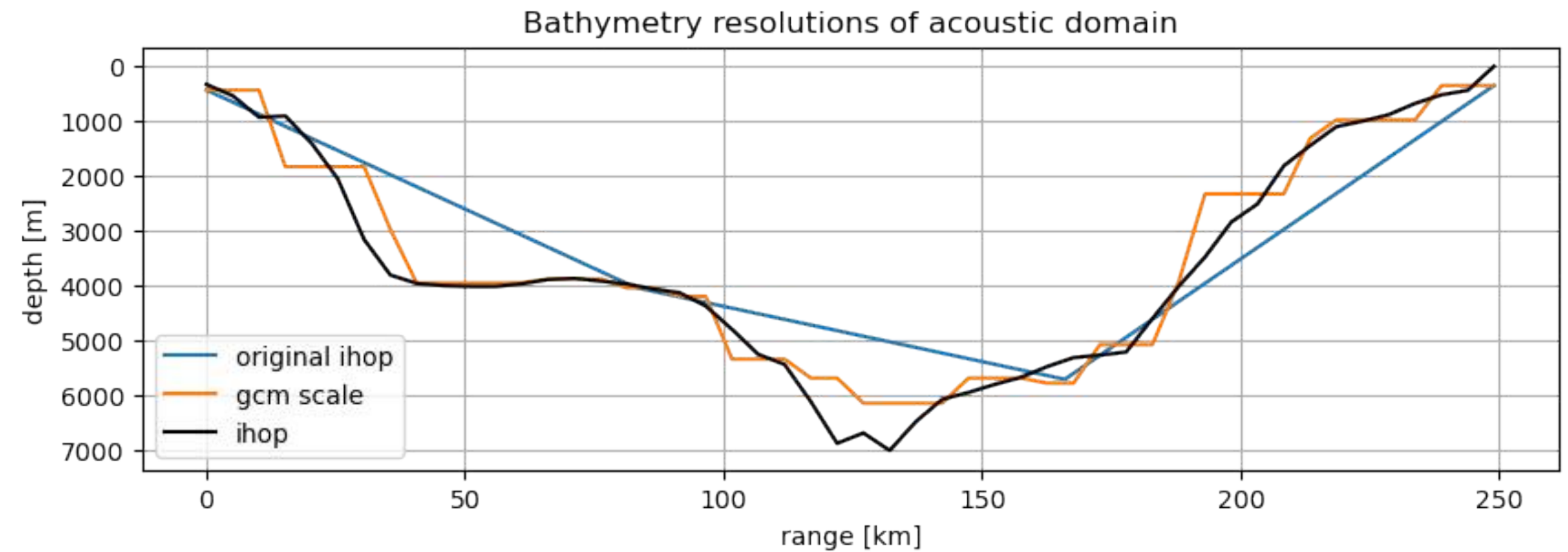
## Acoustic model summary

### MITgcm package, ihop, set-up

- Solver: Ray tracing
- Launch angles:  $\alpha \in [-70, 70]^\circ$ ,  $\Delta\alpha = 0.01^\circ$
- Eigenrays: Geometric Gaussian spreading
- Eigenray declination span:  $\alpha \in [-36.4, 36.4]^\circ$

### Synthetic experiment

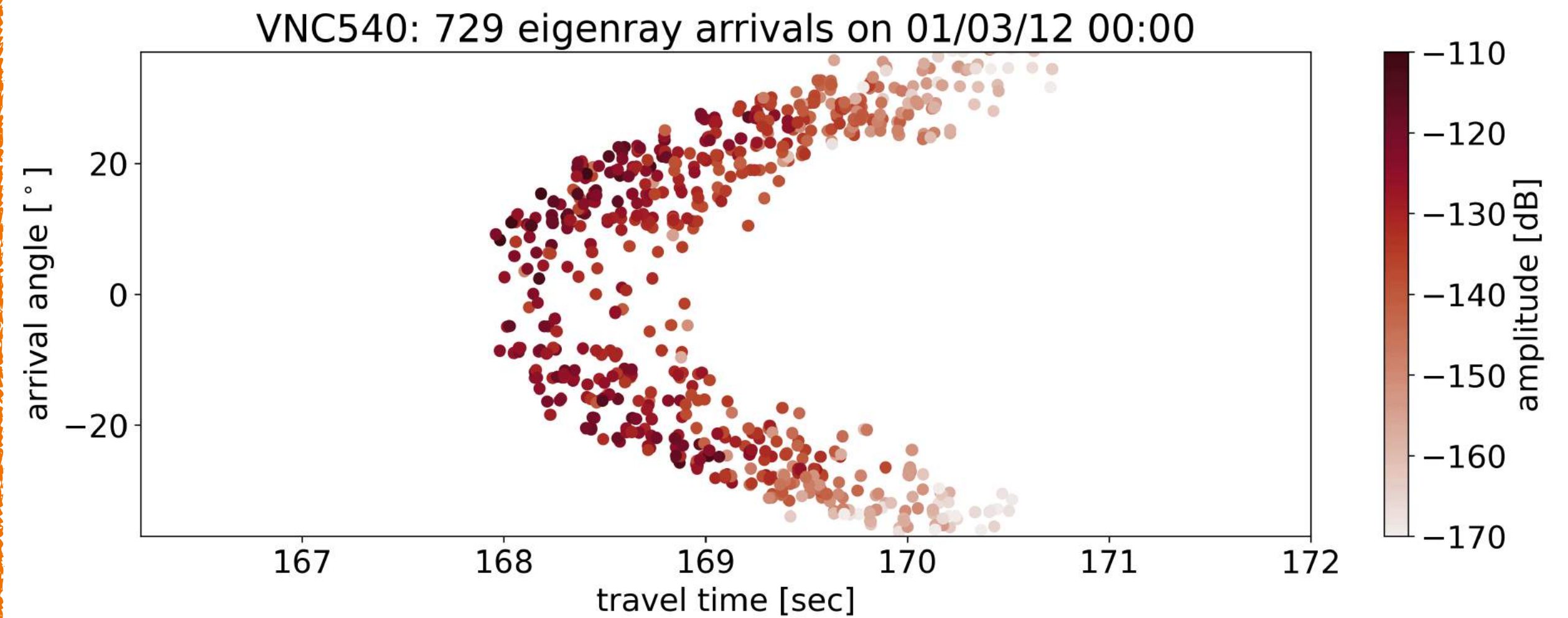
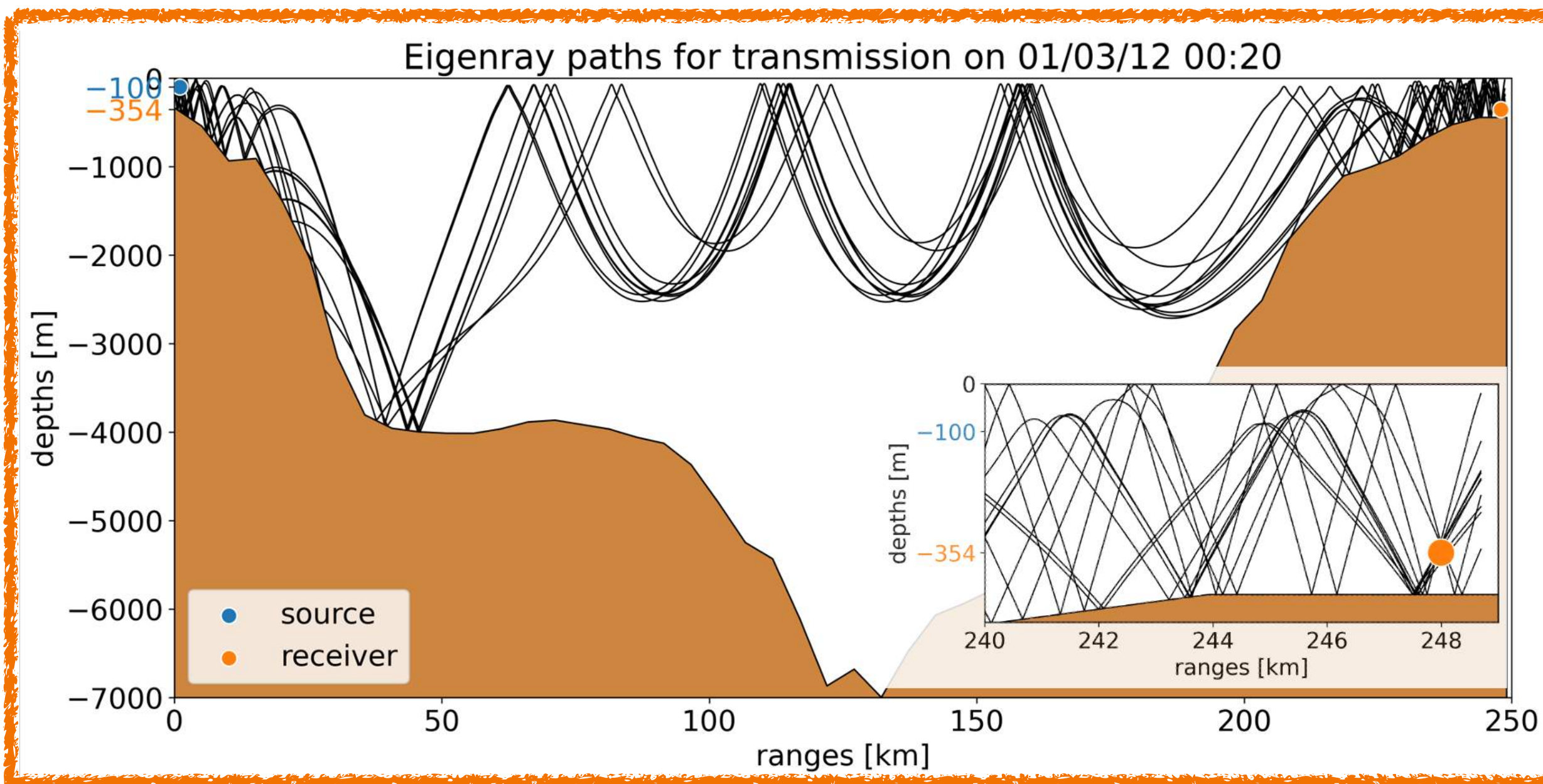
- Time span: 01 March 2012 - 08 March 2012
- Cycle: 1 signal transmission every 10 minutes
- Frequency: 550 Hz



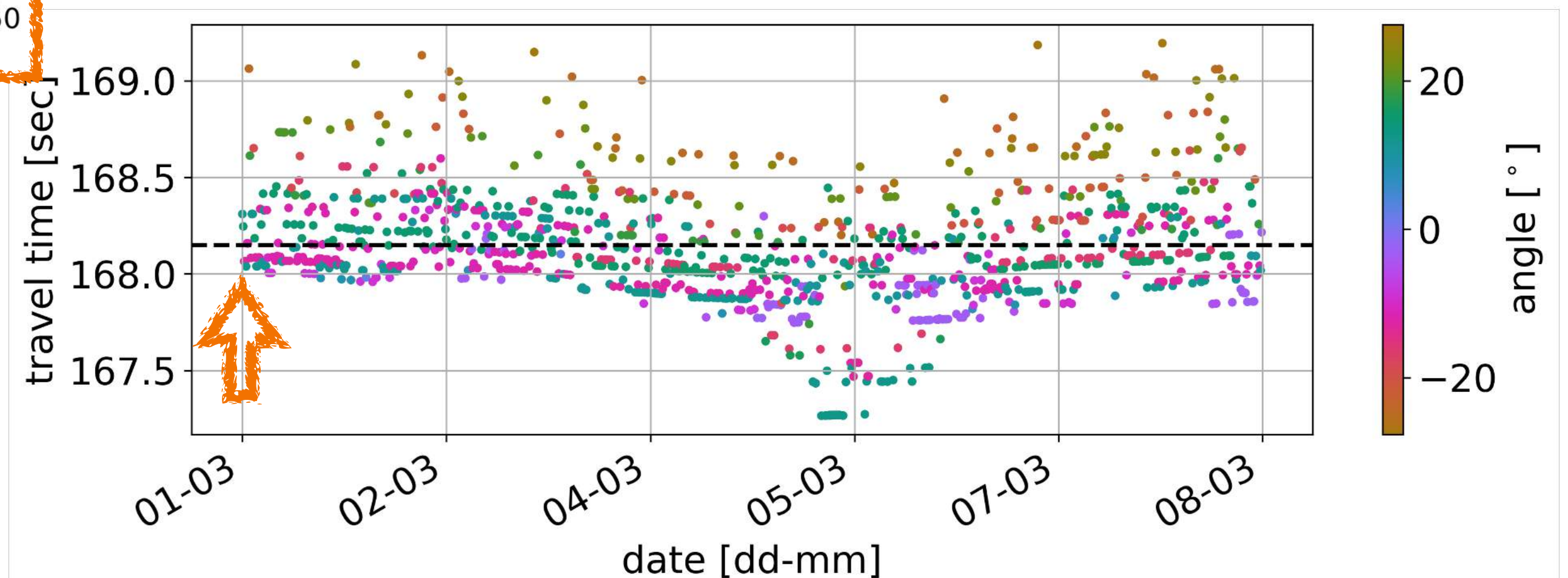


# Case study: Vanuatu/New Caledonia

## A time series of 1080 transmissions from MITgcm



- Parameter-to-observable map can be designed around peak matching in areas with stable arrival structures

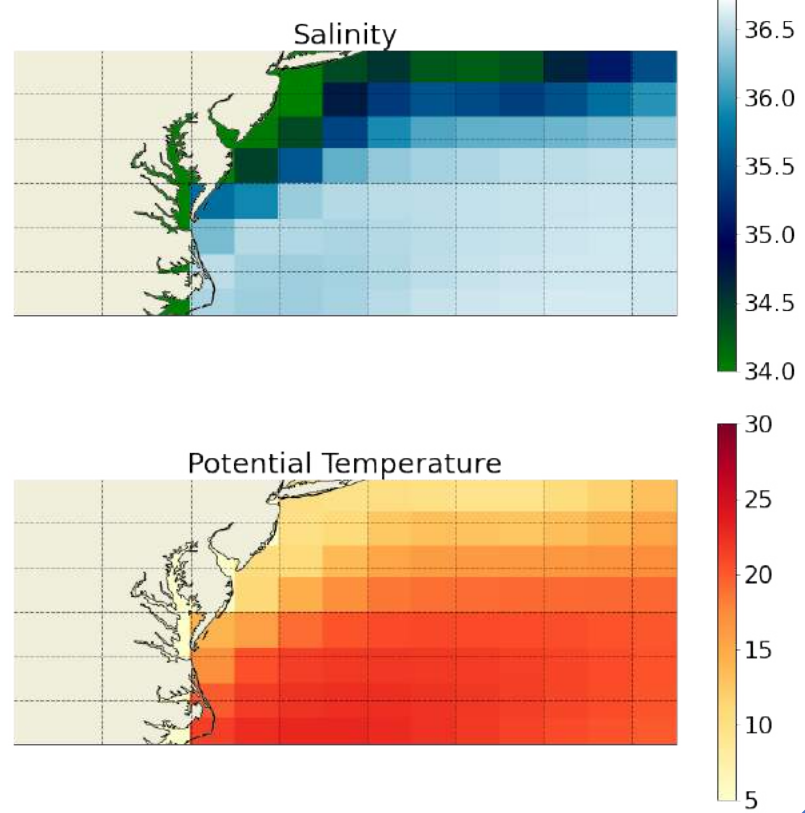




# Introducing acoustics into MITgcm

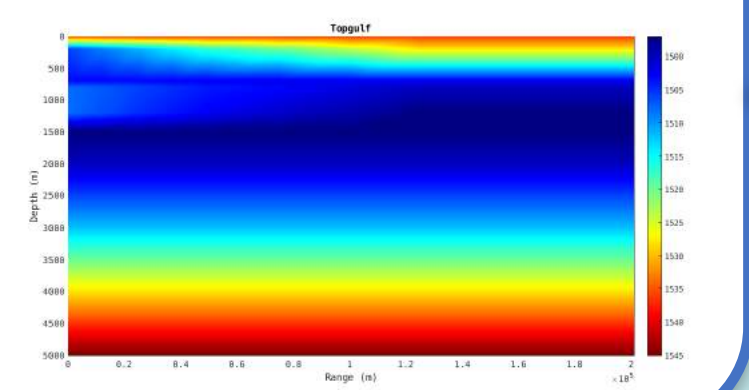
## Adjoint development

MITgcm ocean state



ihop package

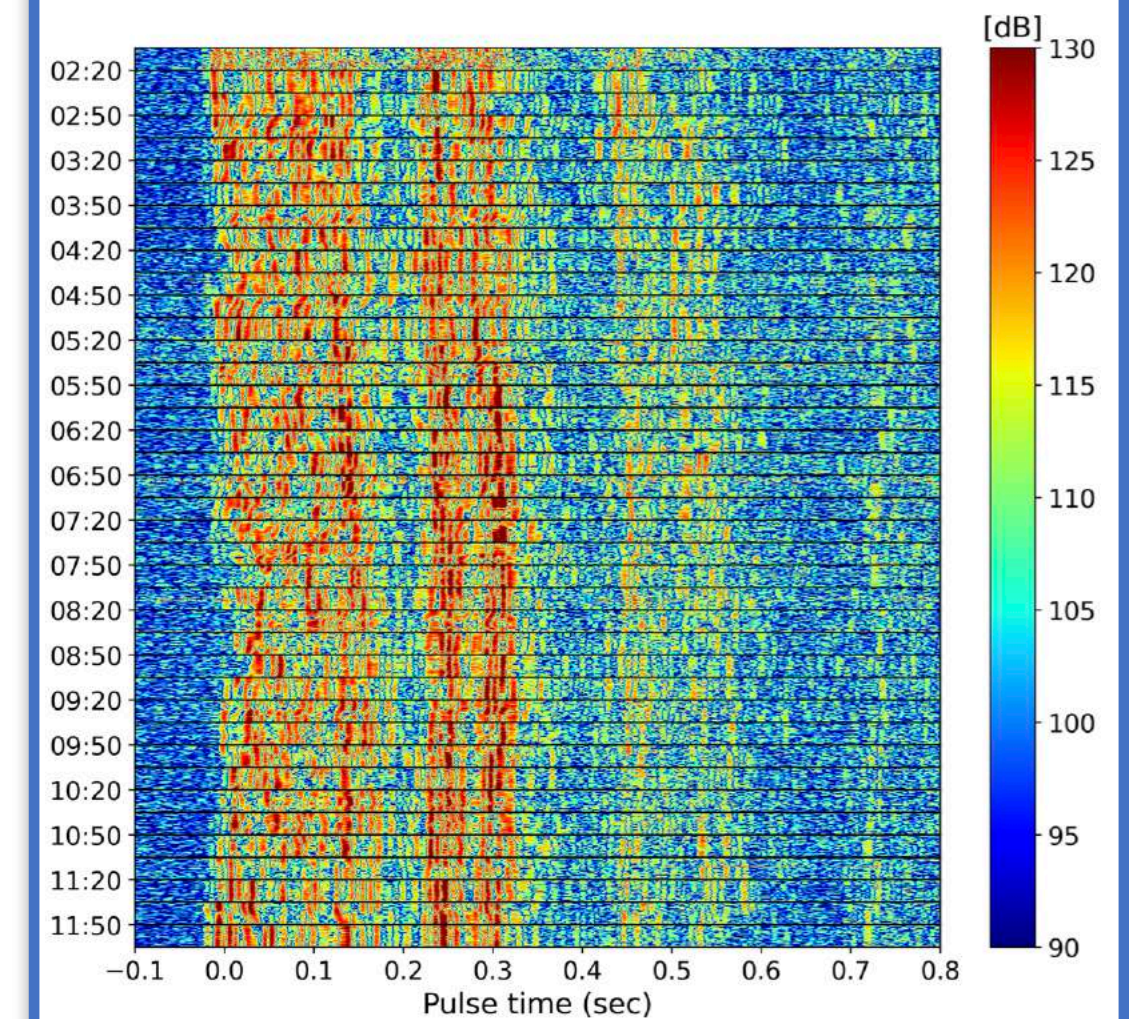
Range dependent SSP



ray trace model

eigenray times of arrival,  $\tau$

Data: Receiver time series



TFO NESBA 2021

Observed times of arrival

$\tau_{\text{obs}}$

User Input:

source(s), receiver(s),  
launch angles, fine-scale bathymetry, ...

$$\frac{dJ}{dT}, \frac{dJ}{dS}$$

$J :=$  cost function



# Acoustic adjoint data assimilation

## Summary of deterministic inversion

- Objective function  $J(\mathbf{s}(\mathbf{m}), \mathbf{m}) : \mathbb{R}^{N_s} \times \mathbb{R}^{N_m} \rightarrow \mathbb{R}$  to be minimized

$$\min_{\mathbf{m}} J(\mathbf{s}, \mathbf{m}) := \min_{\mathbf{m}} \left( J_0(\mathbf{s}, \mathbf{m}) - \sum_{t=t_1}^T \boldsymbol{\mu}^T [\mathbf{s}(t) - \mathbf{M}\mathbf{s}(t-1)] \right),$$

where  $J_0$  is the model-data misfit and regularization,  $\mathbf{M}$  is a linearized representation of the forward ocean model enforced with Lagrange multipliers,  $\boldsymbol{\mu}$  (bold).

- Here, measurements  $\mathbf{y}_{\text{obs}} \in \mathbb{R}^{N_d}$  and control parameters  $\mathbf{m} \in \mathbb{R}^{N_m}$ , where  $N_m \gg N_d$ .



# Acoustic adjoint data assimilation

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$$\min_{\mathbf{m}} J(\mathbf{s}, \mathbf{m}) := \min_{\mathbf{m}} \left( J_0(\mathbf{s}, \mathbf{m}) - \sum_{t=t_1}^T \mu^T [\mathbf{s}(t) - \mathbf{L}\mathbf{s}(t-1)] \right).$$

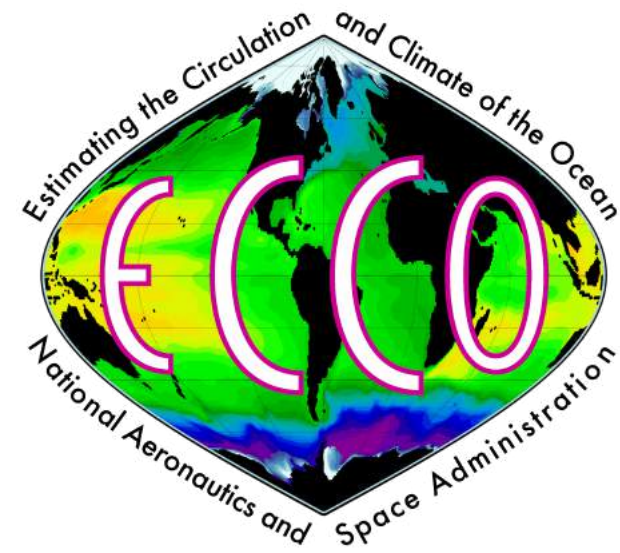
- Now,  $\mathbf{L}(\mathbf{s}(t)) = \mathbf{B}[\mathbf{M}(\mathbf{s}(t-1))]$  includes the acoustic model,  $\mathbf{B}$ , and provides calculated travel times  $\tau \subset \mathbf{s} \in \mathbb{R}^{N_s + N_\tau}$ .



# Summary

## Key points and next steps

- Modeled ray-tracing allows for generation of travel times within a forward ocean circulation run, MITgcm.
  - Method offers simulated underwater acoustics via a *package*, **ihop**.
- **Next steps:** Introduce systematic **acoustic inversion** *within* an ocean circulation state estimate, ECCO.
- Establish a methodology for understanding sensitivities of the oceanic state due to acoustic measurements.



# Thank you!

## Questions?



# References

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**[JM97]:** Marshall, John, et al. "A finite-volume, incompressible Navier Stokes model for studies of the ocean on parallel computers." *Journal of Geophysical Research: Oceans* 102.C3 (1997): 5753-5766.

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**[CC77]:** Chen, Chen-Tung, et al. "Speed of sound in seawater at high pressures." *The Journal of the Acoustical Society of America* 62.5 (1977): 1129-1135.

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**[CW06]:** Wunsch, Carl. "Discrete inverse and state estimation problems: with geophysical fluid applications." *Cambridge University Press* (2006).